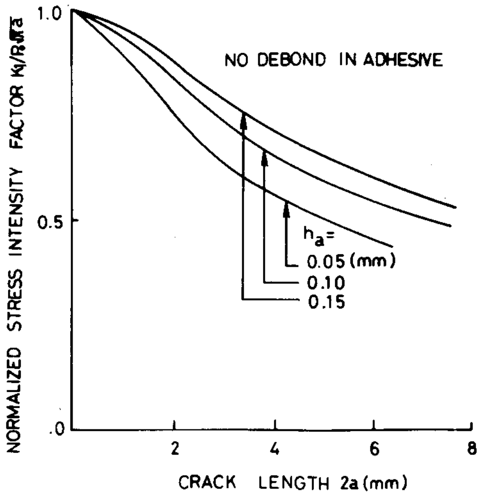

Fig. 4 Influence of adhesive thickness on the stress intensity factors in a two-layer, bonded structure.

In the problem considered here, the opening component K_I is determined by adding the effects of the crack surface pressure P_0 and the body forces X_I and Y_I. The stress intensity factor can be written as

$$\frac{K_I}{\sqrt{a}} + \frac{K_2/\sqrt{a}}{\mu_2} = P_0 + \iint_D [h_I(x_0,y_0)X_I(x_0,y_0) + h_2(x_0,y_0)Y_I(x_0,y_0)]\mathrm{d}x_0\mathrm{d}y_0 \tag{9}$$

In the problem considered here, the shear component of the stress intensity factor K_2 is zero due to the symmetry in loading and geometry.

Numerical Examples

The system of integral equations given by Eq. (6) is the Fredholm type and may be solved by using the standard numerical techniques. In this case, it is done by dividing region D into smaller cells, and unknown functions τ_x and τ_y are assumed to be constant in each cell. Thus, using a numerical integration scheme, the integral equations are reduced to a system of algebraic equations. The kernels in the integral equations have logarithmic singularities, hence the singular part of the kernels is evaluated separately, in closed form. In the actual integration a telescopic grid is used. The cell size is kept small above the crack surface for a distance of about a half-crack length, as the shear stresses are high in this region and are maximum at the boundary of debond. The cell size is progressively increased. The debonded region is approximately represeted by a straight line for integration purpose. The boundary of the domain of integration goes to infinity, hence the size of region D is restricted in numerical analysis such that the stress intensity factors are not appreciably affected.

The convergence of the solution also depends on the crack length. If h_aG_a is very small or the half-crack length is large, the solution will not converge and the shear stresses will oscillate. This can be avoided by decreasing the cell size further and increasing the integration area.

Stress intensity factors for mode 1 (opening mode) and shear stress distribution on the adhesive layer were calculated. The program is fully capable of studying generally othotropic plates and obtaining values of K_I and K_2.

First, stress intensity factors for the isotropic plate were calculated to test the program in special case. The characteristic roots for an isotropic plate are equal for isotropic material, so the complex constant A_k becomes infinite. To avoid this computational problem with this program, the characteristic roots $1.1i$ and $0.9i$ were used. These results are plotted in Fig. 2. These results agree well with the results of Ref. 6 within 6% discrepancy. In the problem of orthotropic plates, adherends and adhesive are boron-epoxy and epoxy, respectively, and their properties are: $E_{Ix} = E_{2x} = 24$ GPa, $E_{Iy} = E_{2y} = 223$ GPa, $V_{yx} = 0.23$, $G_{xy} = 8.4$ GPa, $h_I = h_2 = 2.3$ mm, $Ga = 113.8$ MPa, and $h_a = 0.1$ mm. The influence of debonding size on the stress intensity factors for various crack lengths is shown in Fig. 3. These stress intensity factors have been obtained for no debond and elliptical debond. The end of the major axis of the debond is assumed to coincide with the leading edge of the crack. It is seen that an increase in debonding size increases the stress intensity factors due to less load transfer to the sound layer. The variation of the stress intensity factors with the crack length $2a$ for the various values of the adhesive thickness h_a is shown in Fig. 4.

Conclusions

A mathematical method is outlined to crack problems in adhesively bonded orthotropic structures. The mathematical method of analysis is very useful in analyzing crack problems in a two-orthotropic-ply, adhesively bonded structure; as it is in a two-isotropic-ply, adhesively bonded structure. A debond in the adhesive will cause less load transfer to the sound layer; hence an increase in the stress intensity factors. An increase in adhesive thickness, or a redution in shear modulus, causes less load transfer to the sound layer, resulting in an increase in the stress intensity factors.

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The Affine Equivalence of Local Stress and Displacement Distributions in Damaged Composites and Batdorf's Electric Analog

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Nomenclature

a,b,c = analog device dimensions
a_I,b_I,c_I = composite solid dimensions

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A = area, affine stretch parameter
B,C = affine stretch parameters
E = Young's modulus
E' = Young's modulus for transverse displacements zero ($u=v=0$)
F,Z = geometrical ratios
G = shear modulus
w = axial displacement
x,y,z = analog device positions
x_1,y_1,z_1 = composite solid positions
x_0,y_0,z_0 = affine space positions
α,β = aspect ratios
σ_z = normal stress
τ_{zx},τ_{zy} = shear stress
ρ = resistivity
ϕ = electric potential

Subscripts

e = electrolyte
f = fiber
m = matrix
r = rod

Introduction

RECENTLY Batdorf[1] presented an improved electric analog relating to the title problem. (See Ref. 1 for a listing of related papers.) In the Discussion he remarks, referring to the analog, "However it has in common with numerical calculations of all kinds (including finite element methods) the drawback that *each use of it applies to a single combination of the relevant parameters* and to a single damaged state, so that the total number of combinations of potential interest is enormous" (italics added). The intent of this Note is to show that selected affine transformations make the electric analog much more attractive to use *by making the affine governing/analog equations independent of all elastic and electric constants.*

Analysis

Letting x, y, z and x_1, y_1, z_1 be the position vector components of the analog device and composite solid, respectively, one has, following Batdorf,[1] the analog equations in terms of the rod electric potential and the electrolyte potential,

$$\frac{d^2\phi_r}{dz^2} = \frac{\rho_r}{\rho_e A_r}\oint_r \left(\frac{\partial\phi_e}{\partial y}dx + \frac{\partial\phi_e}{\partial x}dy\right) \tag{1}$$

$$\frac{\partial^2\phi_e}{\partial x^2} + \frac{\partial^2\phi_e}{\partial y^2} + \frac{\partial^2\phi_e}{\partial z^2} = 0 \tag{2}$$

and the governing equations in terms of the fiber axial displacement and the matrix axial displacement,

$$\frac{d^2 w_f}{dz_1^2} = \frac{G_m}{E_f A_f}\oint_f \left(\frac{\partial w_m}{\partial y_1}dx_1 + \frac{\partial w_m}{\partial x_1}dy_1\right) \tag{3}$$

$$\frac{\partial^2 w_m}{\partial x_1^2} + \frac{\partial^2 w_m}{\partial y_1^2} + \frac{E'_m}{G_m}\frac{\partial^2 w_m}{\partial z_1^2} = 0 \tag{4}$$

The accompanying boundary conditions for a variety of cases are discussed by Batdorf[1] and will not be repeated here.

Transforming both sets of the above equations into a *common* affine space via the relations for analog device transformations

$$x = Cx_0 \tag{5}$$

$$y = Cy_0 \tag{6}$$

$$z = Cz_0 \tag{7}$$

and composite solid transformations

$$x_1 = Bx_0 \tag{8}$$

$$y_1 = By_0 \tag{9}$$

$$z_1 = Az_0 \tag{10}$$

where

$$A^2 = (E_f/G_m)A_f \tag{11}$$

$$B^2 = (E_f/E'_m)A_f \tag{12}$$

$$C^2 = (\rho_e/\rho_r)A_r \tag{13}$$

one finds the equivalents of Eqs. (1-4), in a particularly useful affine space, to be

$$\frac{d^2\phi_r}{dz_0^2} = \oint_r \left(\frac{\partial\phi_e}{\partial y_0}dx_0 + \frac{\partial\phi_e}{\partial x_0}dy_0\right) \tag{14}$$

$$\frac{\partial^2\phi_e}{\partial x_0^2} + \frac{\partial^2\phi_e}{\partial y_0^2} + \frac{\partial^2\phi_e}{\partial z_0^2} = 0 \tag{15}$$

and

$$\frac{d^2 w_f}{dz_0^2} = \oint_f \left(\frac{\partial w_m}{\partial y_0}dx_0 + \frac{\partial w_m}{\partial x_0}dy_0\right) \tag{16}$$

$$\frac{\partial^2 w_m}{\partial x_0^2} + \frac{\partial^2 w_m}{\partial y_0^2} + \frac{\partial^2 w_m}{\partial z_0^2} = 0 \tag{17}$$

One notes that Eqs. (14-17) are *independent* of elastic and electric material constants and that there are direct† correspondences between

$$\phi_r \quad \text{and} \quad w_f \tag{18}$$

and

$$\phi_e \quad \text{and} \quad w_m \tag{19}$$

The *derivative* correspondences will be developed shortly.

The geometrical relations between the analog device and composite solid are given by

$$x_1 = Fx \tag{20}$$

$$y_1 = Fy \tag{21}$$

$$z_1 = Zz \tag{22}$$

where

$$F^2 = (E_f/E'_m)(A_f/A_r)(\rho_r/\rho_e) \tag{23}$$

and

$$Z^2 = (E_f/G_m)(A_f/A_r)(\rho_r/\rho_e) \tag{24}$$

and it is noted that $(Z/F)^2 = E'_m/G_m$. The stresses at x_1,y_1,z_1 corresponding to the x,y,z positions in the analog device, as governed by Eqs. (20-22), are given by

$$\sigma_{zf} = \left(\frac{E_f}{Z}\right)\frac{d\phi_r}{dz} \tag{25}$$

†Note that there is an implicit conversion factor of volts/meter (V/m) in all correspondences.

$$\sigma_{zm} = \left(\frac{E'_m}{Z}\right)\frac{\partial \phi_e}{\partial z} \tag{26}$$

$$\tau_{zxm} = \left(\frac{G_m}{F}\right)\frac{\partial \phi_e}{\partial x} \tag{27}$$

$$\tau_{zym} = \left(\frac{G_m}{F}\right)\frac{\partial \phi_e}{\partial y} \tag{28}$$

Furthermore, letting the analog device dimensions be (a,b,c) and letting the composite solid dimensions be (a_I,b_I,c_I), all aspect ratio correspondences may be found by combinatorial division of the following expressions:

$$a_I = Fa \tag{29}$$

$$b_I = Fb \tag{30}$$

$$c_I = Zc \tag{31}$$

e.g., $a_I/b_I = a/b$ and $b_I/c_I = (F/Z)b/c$. It is further convenient to let the analog device aspect ratios a/c and b/c be represented as

$$a/c = \alpha \tag{32}$$

and

$$b/c = \beta \tag{33}$$

Using Eqs. (29-33) one finds

$$a_I/c_I = (F/Z)\alpha \tag{34}$$

$$b_I/c_I = (F/Z)\beta \tag{35}$$

and

$$a_I/b_I = \alpha/\beta \tag{36}$$

Thus in order to predict results over large material and geometry ranges for the composite solid (for a given damage configuration), one must be able to change α and β as easily as possible (once again note that $(F/Z)^2 = E'_m/G_m$). To this end the electrolyte depth can control b_I, and a_I can be controlled by constructing the analog device such that the width a can be easily adjusted (it may be more convenient to adjust c rather than a). It also appears that the easiest way to control F and Z is to vary the electrolyte resistivity. Other ingenious experimental setups may permit more efficient variation of other primitives.

Summary

It has been shown that

1) The composite solid/analog device *affine* equations are elastically and electrically *material independent*.

2) The analog experimental solutions for a given damaged state and given boundary conditions can yield the stress and displacement fields of the composite solid for a wide range of physical variables.

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Errata: "An Alternating Method for Analysis of Surface-Flawed Aircraft Structural Components"

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THE information contained in the footnote on page 749 of this article is incorrect. The footnote should read: "Presented as Paper 82-0742 at the AIAA/ASME/ASCE/AHS 23rd Structures, Structural Dynamics, and Materials Conference, New Orleans, La., May 10-12, 1982."

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